HYDRAULIC ANALYSIS of BASEFLOW and BANK STORAGE in ALLUVIAL STREAMS

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Abstract

This paper presents analytical solutions, which describe the effect of time-variable net recharge (net accretion to water table) and bank storage in alluvial aquifers on the sustenance of stream flows during storm and inter-storm events. The solutions relate the stream discharge, stream-bank sediments water flux and volumes exchange, to the stream inflow hydrograph and groundwater recharge (negative in the case of net abstraction) events, via convolution integrals in terms of the impulse response and unit-step response functions. Discrete kernels can be derived from the continuous-time convolution integrals to predict the stream-aquifer interactions, stream outflow discharges, and cumulative discharge volumes in response to complex discrete-time inflow and groundwater recharge hydrographs. Application to a hypothetical stream-aquifer system shows that the response of the stream discharge to groundwater recharge is a long-term process when compared to increased stream inflows with and without bank storage. The time scale for a steady state stream discharge due to a sustained groundwater recharge is much longer than that due to bank storage releases only, as the unit step response function indicates. Simulations also illustrate the impact of sustained recharge on baseflow and the modification of the stream-aquifer flux exchange and bank storage of surface water.

Introduction

Ground-water/surface water interactions in riparian streams have important hydrologic and ecological implications. Bank storage provides a partial relief to elevated stream stages during storm events, and in combination with ground-water recharge, may sustain base flow during prolonged inter-storm periods, and supplies moisture for aquatic organisms and riparian vegetation. Base flow is that part of total streamflow derived from ground-water discharge rather than from storm runoff.

Numerous numerical and analytical solutions have been developed in the literature for the solution of the bank storage problem in alluvial streams (e.g., *Pinder and Sauer*, 1971; *Moench et al.*, 1974; *Hunt*, 1990; *Serrano and Workman*, 1998; *Zlotnik and Huang*, 1999; *Moench and Barlow*, 2000; *Harada et al.*, 2000; *Hantush et al.*, 2000). However, these methods, particularly the analytical solutions, ignore the effect of groundwater recharge and therefore fail to account

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for baseflow during inter-storm periods. This paper extends the work of *Hantush et al*. (2000) and *Harada etal*. (2000) to address the effect of groundwater recharge as well as bank storage on stream-bank sediments interactions during and after periods of increased stream flows. The Muskingum hydrologic routing method (e.g., *Chow et al*., 1988) is modified to include the effect of bank storage and aquifer recharge on baseflow. The concept of the impulse response and unit step response functions, which is applicable to linear systems, is utilized to relate channel outflow rates, stream-aquifer flow rates, and storage volumes in the bank sediments, to stream inflow and groundwater recharge hydrographs. The coupled boundary value problem for the stream-aquifer system is solved using the Laplace transforms to derive the stream and aquifer impulse response and unit step response functions. The analysis assumes a homogeneous aquifer in each stream reach, of semi-infinite extent, and lateral one-dimensional groundwater flow.

Stream Flow Model

The flow in a stream reach of length L in hydraulic contact with aquifer sediments can be described by the continuity equation (Fig. 1):

$$\frac{dS(t)}{dt} = I(t) - O(t) - 2\,Q(t) \tag{1}$$

in which $S(t)$ = stream reach storage [L^3]; $I(t)$ = stream reach inflow rate [L^3/T]; $O(t)$ = stream reach outflow rate [L^3/T]; and $Q(t)$ = groundwater losses to (or losses from) the bank sediments on one side of the stream [L^3/T]. We use the Muskingum linear storage model (*Chow et al*., 1988),

$$S(t) = \eta\left[\xi\, I(t) + (1-\xi)\, O(t)\right] \tag{2}$$

in which η = the storage time constant for the reach [T]; ξ = a weighting factor that varies from 0 to 0.5. In natural streams, ξ varies from 0 to 0.3 and averages about 0.2. The storage time constant, η, is approximately the kinimatic wave travel time through the reach. If ξ = 0, $S(t)$ = η $O(t)$, the problem is reduced to level pool or reservoir routing. In (2), the stream reach storage, S, in a stream channel is expressed as a combination of a wedge storage, η ξ ($I - Q$), and a prizm storage, η Q .

The channel storage $S(t)$ is related to the average stream stage through the relationship

$$S(t) = W\, L\, H(t) \tag{3}$$ ✪

in which W = the average stream width [L]; and $H(t)$ = the average stream stage [L] (Fig. 1).

✪ Equation (3) is a correction to Eq. (3) of Hantush et al. (2000)

Aquifer Model:

In this analysis, the aquifer is unconfined, of semi-infinite extent, and homogeneous. Groundwater flow is assumed to be one-dimensional and lateral to the stream reach, and the aquifer is separated from the stream by a semipervious layer of sediments. The flow in this aquifer may be described by the linearized Boussinesq partial differential equation:

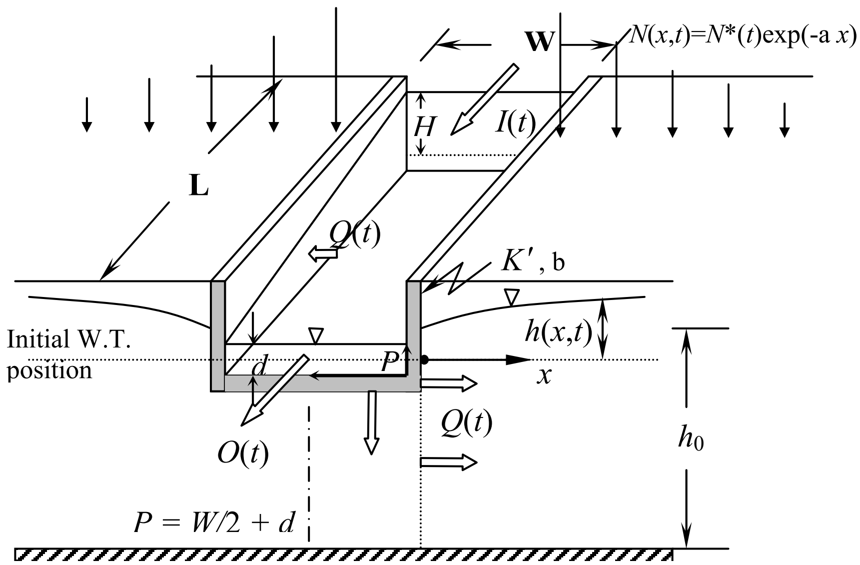

Fig. 1 Illustration of a stream hydraulically connected to an aquifer

$$\frac{\partial h(x,t)}{\partial t} = D\frac{\partial^2 h(x,t)}{\partial x^2} + \frac{1}{S_y}N(x,t) \tag{4}$$

subject to the initial and boundary conditions:

$$h(x,0) = 0 \tag{5}$$

$$-T\frac{\partial h(0,t)}{\partial x} = P K'\frac{[H(t) - h(0,t)]}{b} = \frac{1}{L}Q(t) \tag{6}$$

$$h(\infty,t) = 0 \tag{7}$$

in which $h(x,t)$ = the water-table fluctuations relative to the initial equilibrium position [L]; $N(x,t)$ = net recharge rate (or net evapotranspirative losses in riparian buffers) [L/T]; $D = T/S_y$ is the aquifer diffusivity [L^2/T]; T = the average transmissivity of the aquifer [L^2/T]; K = the hydraulic conductivity [L/T]; S_y = the specific yield of the unconfined aquifer; $Q(t)$ = the stream-aquifer flux exchange within channel length L [L^3/T]; P = half the wetted perimeter of the stream (Fig. 1) [L]; K'= the hydraulic conductivity of the stream bed [L/T]; and b = the thickness of the stream bed [L/T].

In Eq. (4) the water-table fluctuations are assumed to be small relative to the average saturated thickness of the aquifer. The third-type boundary condition (6) ignores storage in the alluvial sediments below the stream; i.e., at any point in time, groundwater flux at $x = 0$ is balanced by the discharge to the stream across the streambed sediments. *Zlotnik and Huang* (1999), however, investigated the effect of storage below the stream on the aquifer hydraulic head.

Uniform Net Recharge in Riparian Buffers

This analysis considers exponentially decreasing recharge laterally from the stream (Fig. 1), $N(x,t) = N^*(t)\exp\{-a\,x\}$, where $N^*(t)$ = recharge rate (or losses) per unit area at the stream-aquifer interface [L/T]; and a = the spatial rate of decrease of recharge rate [L^{-1}]. A uniformly distributed recharge over a finite lateral distance can be represented equivalently by the exponential model. Lets consider the case of an average uniform recharge of magnitude $N^*(t)$ distributed over a distance l (e.g., width of a riparian buffer). By equating the area under the uniform recharge with that under the equivalent exponential model (Fig. 1), we have $N^*(t)\,l = \int_0^\infty N^*(t)\,e^{-a\,x}dx$, and one can easily show that $a = 1/l$. That is, the equivalent recharge decreases at a rate equal to the reciprocal of the recharge buffer width. This same concept can be applied to more complex recharge distributions. While such representation of exponential recharge model conserve flow rates, it may not yield accurate representation of the hydraulic head distribution in the aquifer. The latter, however, is not the focus of this effort.

Recharge Impulse Response Function

The stream is assumed to be initially in a state of equilibrium with the aquifer and recharge. Thus, the variables $I(t)$, $O(t)$, $h(x,t)$, $N^*(t)$, and $H(t)$ in Eqs. (1-4) are defined relative to their initial values (i.e., $I(0) = O(0) = 0$, $N^*(0) = 0$, $h(x,0) = 0$, and $H(0) = 0$). The application of the Laplace transforms to Eqs. (1-7) should yield the following algebraic relationships in the Laplace domain:

$$\widetilde{S}(p) = \eta\left\{\xi\,\widetilde{I}(p) + (1-\xi)\,\widetilde{O}(p)\right\} \tag{8}$$

$$p\,\widetilde{S}(p) = \widetilde{I}(p) - \widetilde{O}(p) - 2\,\widetilde{Q}(p) \tag{9}$$

$$\widetilde{S}(p) = W\,L\,\widetilde{H}(p) \tag{10}$$

$$\widetilde{O}(p) = \widetilde{u}(p)\,\widetilde{I}(p) + \widetilde{u}_n(p)\,\widetilde{N}^*(p) \tag{11}$$

$$\widetilde{Q}(p) = \widetilde{u}^*(p)\,\widetilde{I}(p) + \widetilde{u}_n{}^*(p)\,\widetilde{N}^*(p) \tag{12}$$

where

$$\widetilde{u}_n = \frac{2\,L\,D}{(1-\xi)\,\eta}\,\widetilde{\Pi}(p) \tag{13a}$$

$$\widetilde{\Pi}(p) = \frac{1}{R\,p^2 + (R\,a+1)\sqrt{D}\,p^{3/2} + (a\,D + \gamma\,R)\,p + \sqrt{D}\left[a\gamma R + 1/(\eta(1-\xi))\right]\sqrt{p} + aD/(\eta(1-\xi))} \quad (13b)$$

In which the Laplace transform of a function $f(t)$ is defined as $\widetilde{f}(p) = \int_0^\infty f(t)\,e^{-pt}\,dt$. In Eq. 13(b) $\gamma = 1/(\eta(1-\xi)) + (2\,P/W)(K'/b)$, and $R = T\,b/P\,K'$ is the retardation coefficient [L] modified for partial penetration. $R \approx K\,b/K'$ for completely penetrated aquifer .

After substituting (8), (11), and (12) into (9) and equating the coefficients of $\widetilde{I}(p)$ and $\widetilde{N}^*(p)$, one obtains

$$\widetilde{u}_n^{\,*}(p) = -\frac{1}{2}\{\eta(1-\xi)\,p + 1\}\,\widetilde{u}_n(p) \quad (14)$$

The inverse Laplace transforms of (11) and (12) is given by the following convolutions:

$$O(t) = \int_0^t u(t-\tau)\,I(\tau)\,d\tau + \int_0^t u_n(t-\tau)\,N^*(\tau)\,d\tau \quad (15)$$

$$Q(t) = \int_0^t u^*(t-\tau)\,I(\tau)\,d\tau + \int_0^t u_n^{\,*}(t-\tau)\,N^*(\tau)\,d\tau \quad (16)$$

in which $u(t)$ = impulse response function [T^{-1}], which describes the incremental distribution in time of the channel (or stream reach) outflow rate in response to an instantaneous input of unit amount at t = 0 at the upstream inflow boundary (*Chow et al.*, 1988); $u_n(t)$ = impulse response function [L^2/T], which describes the incremental distribution in time of the channel outflow rate in response to an instantaneous recharge of unit amount at t = 0 at the water table; $u^*(t)$ and $u_n^*(t)$ are defined similarly for groundwater flux at the stream-bank sediments interface. The impulse response functions $u(t)$ and $u^*(t)$ (bank storage effect) were obtained by *Hantush et al*. (2000, 2001),

$$u(t) = \frac{1}{(1-\xi)^2\,\eta}\,\frac{4}{\pi}\,\frac{T\sqrt{D}}{W}\int_0^\infty \frac{y^2\,e^{-y^2 t}}{\Lambda 1(y)}\,dy - \frac{\xi}{1-\xi}\,\delta(t) \quad (17)$$

$$u^*(t) = \frac{2\,T\sqrt{D}}{W(1-\xi)\,\pi}\int_0^\infty \frac{y^2\,e^{-y^2 t}}{\Lambda 1(y)}\left(y^2 - \frac{1}{\eta(1-\xi)}\right)dy \quad (18)✪$$

✪ Equation (18) is a correction to Eq. (18) of Hantush et al. (2000)

The impulse response functions $u_n(t)$ and $u_n^*(t)$ represents the new elements of this paper as they describe the effect of recharge on baseflow. $u_n(t)$ is derived by inverting $\widetilde{u}(p)$ (13a) in the complex plane:

$$u_n(t) = \frac{2\,L\,D}{(1-\xi)\,\eta}\,\frac{1}{2\,\pi\,i}\int_{\alpha-i\infty}^{\alpha+i\infty}\Pi(z)\,e^{z\,t}\,dz \tag{19}$$

This integral is evaluated by introducing a branch cut along the negative real axis (Fig. 2) with the argument of the principal branch $\sqrt{z}$ defined from -π to π ($z = r\,e^{i\theta}$ and $-\pi \le \theta \le \pi$), then carrying the integration along the contours C_ρ, Γ_1, Γ_ε, Γ_2 to yield

$$u_n(t) = \frac{L}{(1-\xi)\,\eta}\,\frac{4}{\pi}\,D^{3/2}\int_0^\infty \frac{\left\{\left[a\,\gamma\,R + 1/(\eta(1-\xi)\right] - (1 + a\,R)\,y^2\right\}}{\Lambda(y)}\,y^2\,e^{-y^2\,t}\,dy \tag{20a}$$

$$\Lambda(y) = \left[R\,y^4 - (a\,D + \gamma\,R)\,y^2 + \frac{a\,D}{\eta\,(1-\xi)}\right]^2 + y^2\left[\sqrt{D}\left(a\,\gamma\,R + \frac{1}{\eta(1-\xi)}\right) - (a\,R+1)\sqrt{D}\,y^2\right]^2 \tag{20b}$$

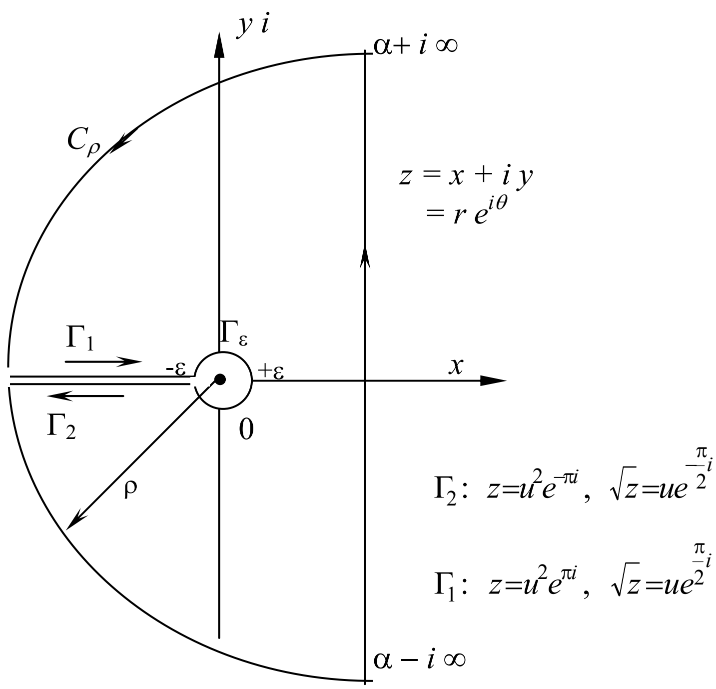

Fig. 2 Integration path lines in the complex plane for the Laplace inverse transformation of $\widetilde{u}_n(p)$.

$u^*_n(t)$ and can be obtained directly from $u_n(t)$ by applying the Laplace inverse transform to both sides of (14) and noting that $u_n(0) = \lim_{p\to\infty} p\,\widetilde{u}_n(p) = 0$,

$$u_n^*(t) = -\frac{\eta(1-\xi)}{2}\frac{du_n(t)}{dt} - \frac{1}{2}u_n(t) \tag{21}$$

The second integrals on the right side of (15) and (16) constitute the new elements of this effort and they describe the groundwater recharge, $N(x,t)$, contribution to baseflow. The first integrals in (15) and (16), however, account for the bank storage effect.

Figure 3 shows stream discharge impulse response and unit step response functions versus time in log-log plots for the following hypothetical stream-aquifer data: $K = 36$ *m*/*h*, $K' = 0.2$ *m*/*h*, $b = 0.5$ *m*, $W = 20$ *m*, $P = 20$ *m*, $T = 720$ m^2/h, $S_y = 0.27$ (i.e., $D = 2667$ m^2/h, R = 90 *m*), $L = 1000$ *m*, $a = 0.01$ m^{-1} (equivalent to a uniform recharge distributed over a distance 100 *m*), $\xi = 0.15$, and $\eta = 0.4$ *h*. The attenuating effect of bank storage is clear on $u(t)$ as it shows smaller values initially when compared to the case of no stream-aquifer interactions, then greater and more persisting values after (for this data) 1.5 *h*, due to bank storage (Fig. 3a). In the case of hydraulically isolated stream, the impulse response function $u(t)$ approaches zero after 3.5 *h* for the given data, and the stream discharge response to an inflow event is therefore relatively immediate. The recharge impulse response $u_n(t)$ in Fig. 3a shows a greater persistence and declines slowly. Although not shown, $u_n(t)$ increases from zero ($u_n(0) = 0$) to a finite peak value at early time and then declines gradually thereafter. On the other hand, $u(0) = 1/[(1-\xi)^2\,\eta] - \delta(0)\,\xi/(1-\xi)$ (*Chow*, 1988, page 271). That is, $u(t)$ increases from $-\infty$ at $t = 0$ to $1/[(1-\xi)^2\eta]$ instantly at $t = 0^+$. The more gradual and persisting response of the stream discharge to an aquifer recharge event may be the result of diffusive flow in the aquifer sediments.

Recharge Step Response Function

The integral of the recharge impulse response function $u_n(t)$ is the unit step response function $g_n(t)$,

$$g_n(t) = \int_0^t u_n(\tau)\,d\tau \tag{22a}$$

The evaluation of this integral after substituting (20a) for $u_n(\tau)$ is straightforward,

$$g_n(t) = \frac{2L}{a} - \frac{L}{(1-\xi)\eta}\frac{4}{\pi}D^{3/2}\int_0^\infty \frac{\{[a\gamma R + 1/(\eta(1-\xi)] - (1+aR)\,y^2\}}{\Lambda(y)}e^{-y^2 t}\,dy \tag{22b}$$

The unit step response function $g_n(t)$ is defined as the incremental stream outflow rate $O(t)$ response to a unit step increase of the recharge rate $N^*(t)$ from 0 to 1 at time 0 (*Chow et al.* 1988). It can also be defined as the incremental response of the cumulative stream outflow volume (i.e., integral of the outflow rate from time 0 to t) at a later time due to a unit amount of recharge applied instantly at time 0 (*Hantush et al.*, 2001). It can be shown that

$\lim_{t\to\infty} g_n(t) = 2\,L/a$. That is, after large time, the stream discharge, $O(t)$, would be equal to the unit recharge rate multiplied by the entire area contributing recharge to the stream reach, 2 L/a.

The cumulative stream-aquifer volume exchange is defined as:

$$V(t) = 2\int_0^t Q(\tau)\,d\tau \qquad (23a)$$

in which $V(t)$ = cumulative stream-aquifer flux at time t [L^3]. The use of (16) in (23a) and shifting the order of integration should yield

$$V(t) = 2\int_0^t g^*(t-\tau)\,I(\tau)\,d\tau + 2\int_0^t g^*{}_n(t-\tau)\,N^*(\tau)\,d\tau \qquad (23b)$$

in which $g^*(t)$ = integral of $u^*(\tau)$ from $\tau = 0$ to t, is the unit step response function associated with the inflow hydrograph $I(t)$; it is defined as the incremental stream-aquifer discharge response, $Q(t)$, to a unit step increase of inflow rate $I(t)$ from 0 to 1 at time 0. $g^*{}_n(t)$ = integral of $u^*{}_n(\tau)$ from $\tau = 0$ to t, is the unit response function associated with the recharge process. Note the contribution of each of the bank storage and recharge to cumulative outflow volume is characterized by their corresponding unit step response function, $g^*(t)$ and $g^*{}_n(t)$, respectively; hence, the term incremental in their definitions and those of $u^*(t)$ and $u^*{}_n(t)$ above. The function $g^*(t)$ is given by *Hantush et al*. (2001)

$$g^*(t) = \frac{2T\sqrt{D}}{W(1-\xi)\pi}\int_0^\infty \frac{1-e^{-y^2 t}}{\Lambda 1(y)}\left(y^2 - \frac{1}{\eta(1-\xi)}\right)dy \qquad (24a)$$

where

$$\Lambda 1(y) = R^2\,y^2\,(\gamma - y^2)^2 + D\left[\frac{1}{\eta\,(1-\xi)} - y^2\right]^2 \qquad (24b)$$

and $g^*{}_n(t)$ can be obtained by integrating the right side of 21 from $\tau = 0$ to t,

$$g^*{}_n(t) = -\frac{\eta\,(1-\xi)}{2}u_n(t) - \frac{1}{2}g_n(t) \qquad (25)$$

Figure 3b shows the unit step response of the stream discharge $O(t)$ to stream inflow and groundwater recharge. For each of the two cases of hydraulically isolated (no interactions) and interacting stream-aquifer system (bank storage with no recharge), the unit response function, $g(t)$ (*Hantush et al*., 2001) approaches a steady value of 1 relatively faster than $g_n(t)$, which attains after a much longer time a steady value of 2 L/a = 2 (1000/0.01) = 2 × 10^5 m^2. The unit step response $g(t)$ is initially smaller for interacting stream-aquifer system than for a hydraulically isolated stream, however, approaches the same maximum value of 1 asymptotically.

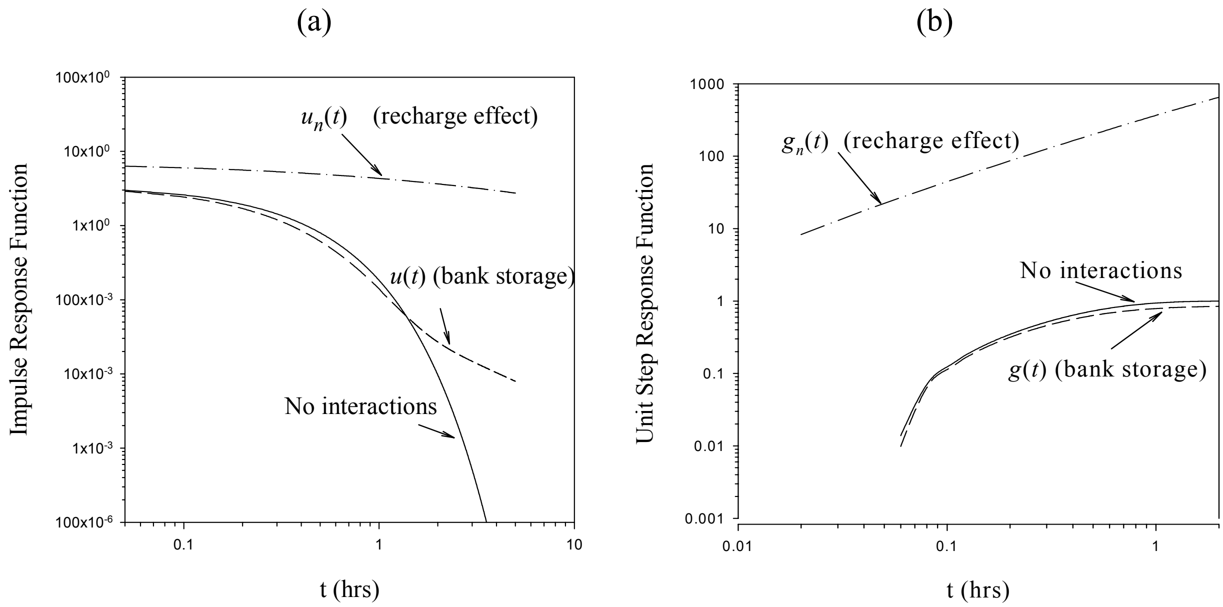

Fig. 3 Logarithmic plots of the stream discharge: (a) impulse response, and (b) unit step response functions for different cases of stream aquifer interactions.

Figure 4 shows simulated results of $O(t)$, $Q(t)$, and $V(t)$ in response to a hypothetical inflow hydrograph and a uniform recharge process occurring over a distance of $l = 100$ m lateral to the stream axis. The inflow hydrograph is assumed to be of the type proposed by *Cooper and Rorabaugh* (1963), $I(t) = \phi\, I^* \, e^{-\delta\, t}\, [1\text{-}\cos(\omega t)]$, when $t_0 \le t \le t_0 + \Delta T$, and $I(t) = 0$, $t \ge t_0 + \Delta T$ and $t \le t_0$; $t_0 = 10\ h$ is the starting time for the storm event, $\Delta T = 24\ h$ is the time duration of the storm; I^* is the peak inflow rate [L^3/T]; $\omega = 2\pi / \Delta T$; $\phi = e^{\delta t_c} / [1 - \cos(\omega t_c)]$; $t_c = 16\ h$ is the time at which peak inflow occurs; and $\delta = \omega \cot(\omega t_c / 2)$. Groundwater recharge rate $N(x,t)$ is assumed to be equal to $10^{-5} \times I^*\ m^{-2}$, $0 \le x \le 100\ m$, and 0, $x > 100\ m$. The equivalent exponential recharge model is $N(x,t) = 10^{-5}\, I^* \exp(-0.01 x)$; $a = 1/l$. Prior to the runoff event, $t < 10\ h$ and $t > 100\ h$, the stream discharge (reach outflow rate), $O(t)$, is sustained entirely by groundwater recharge (i.e., base flow), as Fig. 4(a) indicates. Similarly, the simulated stream-aquifer exchange, $Q(t)$, is also dominated by groundwater discharge due to recharge during the same time periods (Fig. 4 (b)). Bank storage effect is evident during the time period $10 \le t \le 30\ h$. Figure 4(c) shows the volume of surface water $V(t)$ stored in the aquifer. The effect of groundwater recharge is to diminish bank storage as indicated by the negative integrated flux into the aquifer. In this example, all volumes of surface water stored in the bank sediments during the storm event are flushed out and recycled back to the stream by $t = 30\ h$. In the case of no recharge ($N = 0$), bank storage of surface water persists longer.

Hantush et al. (2000, 2001) extend the analysis of Chow et al. (1988) to estimate $O(t)$, $Q(t)$, and $V(t)$ in response to discrete-time stream inflow and recharge events of general shape.

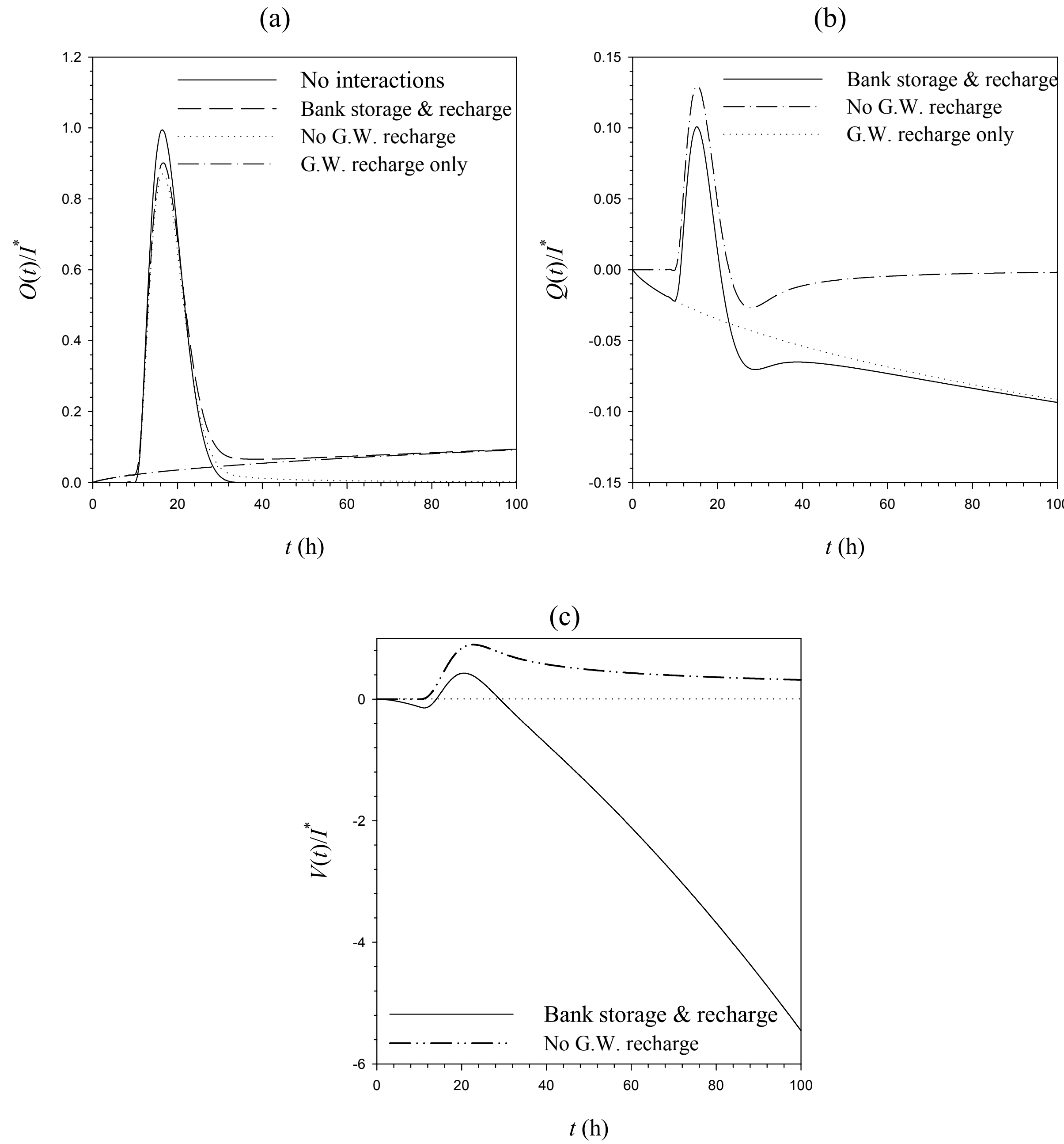

Fig. 4 Simulated results for a hypothetical stream inflow and recharge events with and without groundwater recharge: (a) stream discharge, (b) stream-aquifer groundwater flux, and (c) cumulative volume into the aquifer.

Summary

Analytical solutions were derived that describe the dynamic interactions between a stream and the surrounding aquifer sediments in alluvial environments. The Muskingum method for of stream flow routing was modified for bank storage and groundwater recharge (e.g., accretion to the water table). The solution is capable of simulating baseflow during storm and inter-storm periods. The stream reach discharge and stream-aquifer flux and volume exchange were expressed by convolutions integrals of the stream inflow and groundwater recharge hydrographs, and the corresponding impulse response and the unit step response functions. Analysis based on a hypothetical example showed the discrepancy in the time scale of the response of the stream discharge to potential runoff and groundwater recharge events. The stream discharge response to the recharge process was slower but showed a significant long-term component. This analysis may have implication on the effect of land-use watershed management on stream flows in alluvial environments and water storage in riparian buffers.

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